Using GeoGebra as a problem solving tool

Chris Sangwin

Mathematics Education Centre
Loughborough University

May 2015

What is GeoGebra?

GeoGebra is computer software.

Dynamic mathematics for learning and teaching.

`http://www.geogebra.at`

(Other software is available!)

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A personal journey...

Starting in 2006:

- diagrams for *How Round is Your Circle?* in LaTeX; (Hart's A-frame)
- applets for the website;
- dynamic diagrams in lectures. (Complex numbers)

... and interesting personal insights ...

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GeoGebra is not a linkage tool!

- Tangent to a cubic through the average of two roots.

Use with students

GeoGebra is designed for use with students!

MSM1Y: developing mathematical reasoning

Texan topologist Robert Lee Moore (1882–1974)

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Course setup

- Optional first year module. (“Expected” for MSci)
- A 10–20 students per group.
- Aim is to develop problem solving and raise confidence, not to “cover stuff".
- Set up by Chris Good in 2004
  Point set topology (3 years)
- Taken over by Chris Sangwin in 2007
  Geometry (5 years)
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Basis of Moore's Method

1. Mathematical problems posed to the whole class.
2. Students solve problems independently.
3. Students present their solutions to the class.
4. Students discuss solutions.
5. Students decide if answers are correct.

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Programme structure and assessment

Students take 120 credits per year.
This course is 20 credits, split over two semesters.

- Semester 1 traditional geometry course.
- Semester 2 Moore method course.

- Quality of best 2 presentations 25%
- Individual written solutions to all problems 25%
- (Semester 2 *"Formal Euclidean Geometry"* 50%).

(No exam)
Geogebra was not mandatory, but use was encouraged for presentations.

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Correctness: *“is this a proof?"*

Work is “correct" when everyone (including the teacher)

1. understands the solution,
2. can find no technical or logical mistakes,
3. is confident more detail could be given on request,

and

4. it is complete, in that it correctly identifies all cases.

A long way from copying an expected model answer.

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Structure in problems

The key distinctive feature of Moore Method is *structure in problems*.
I.e. the problems form a connected whole, ideally leading to a major result.
However, *to the students* they initially appear to be *puzzles*.
They may always appear to be puzzles!

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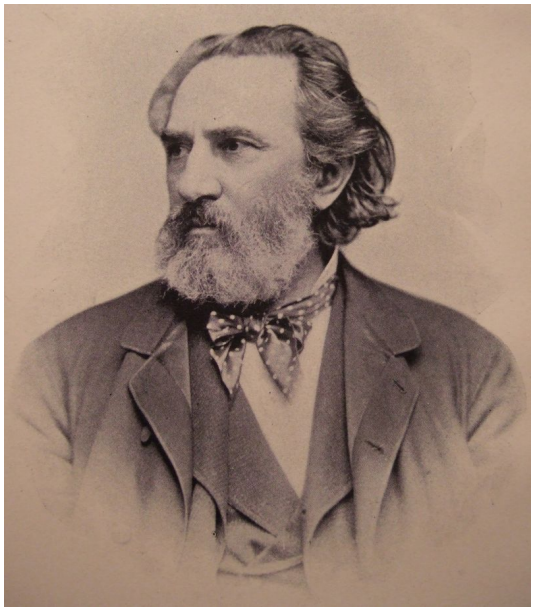
Selecting the problems is key

Moore chose Topology.

Kinematics

The geometry of machine movement.



Franz Reuleaux (1829–1905)

- Kinematics of Machinery, (1876)
- The Constructor, (1904)

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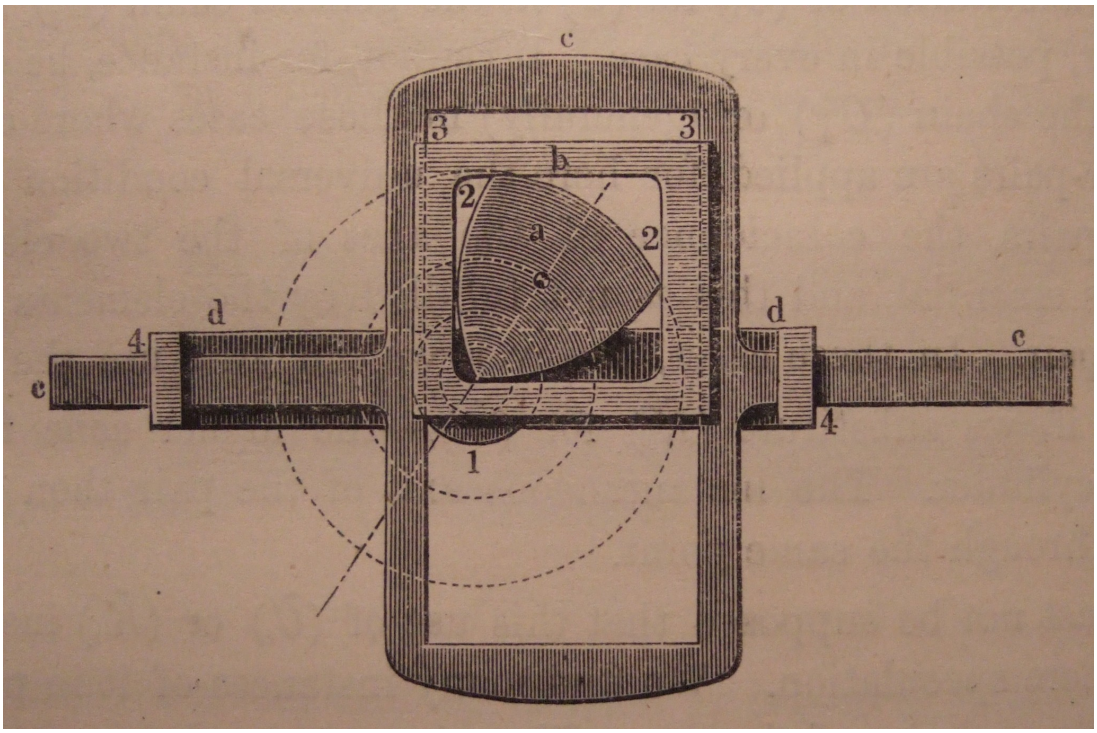
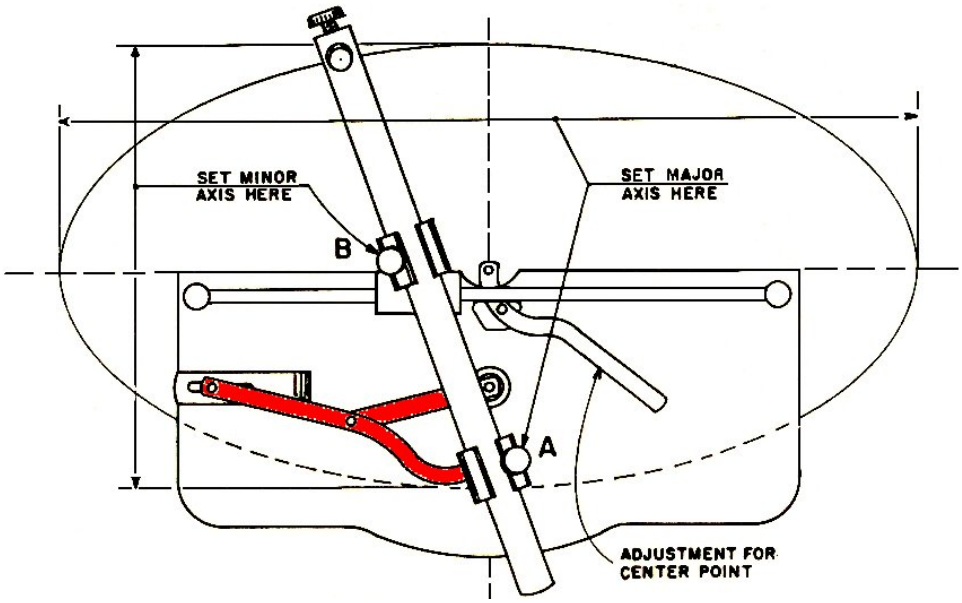
Cam mechanism



Example: the cat on the ladder

0.1 A ladder standing on a smooth floor against a wall slides down to the floor. Along what curve does a cat sitting in the middle of the ladder move?

Asside: grasshopper linkage



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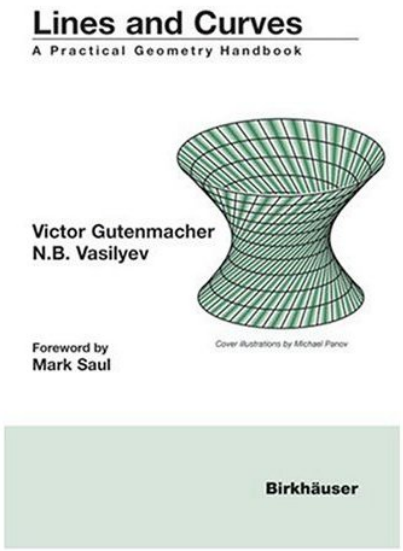
Choice of problems (before 2011!)

Taken from V. Gutenmacher and N.B. Vasilyev. *Lines and curves: a practical geometry handbook*. Birkhauser, 2004.



Reasons

- Geometry
- Locus problems, *kinematics*, (machine motion)
- Modelling aspects
- Little prior knowledge
- Often different solution approaches

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What is proof?

Results: end of course questionnaire 2008
Typical answer:

> *a logical step by step argument that, with no unnecessary steps, that shows without a doubt that the statement is true.*

Has your concepts of proof changed?

Yep! At first my idea was of a proof was writing long chunks of work and simplity hoping the correct proof was in there. As I gained more experience I found a good proof is consise and to the point. Rather then writing pages and pages for a proof with unnessary info I learnt to shorten my work down to say a page with just the nessary info.

that it is not necessarily a long winded argument and the most satisfying proof is a short one.

Yes, I use to feel that the complicated long proofs always were the 'best' i can now appreciate that this is far from true.

Did anything in the course surprise you?

The amount of work and how hard it was.

The workload at the beginning but as I got used to it, it helped me a lot with the other courses especially MSM1B. [i.e. real analysis]

Not sure if it counts as an event, but just the fact that i actually really enjoyed geometry. Before the course i would have said I didn't like it at all, but now i realise this is not ture, and really wish I could do geometry next year.

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Reactions


MARMITE
YEAST EXTRACT
MARMITE
RICH IN B VITAMINS • 100% VEGETARIAN

Not all students engaged...

Overall i think that it is an extremly interesting course and in a way wish i had turned up to more lectures and given it alot more time. When signing up i had no idea how much hardwork it would be and deffinitly sturggled to keep up with the pace of the course. It is obviopusly my fault that i failed and will have to retake it as i failed to turn up to most of the lectures but think that its a tiny bit unfair that people gained 20 credits doing the other maths course which could have easily passed with out paying much attention [...] I think the main probelm is that a momd at birmingham is sort of seen as abit of a joke and easy credits but y is deffinitly not that. [...] Anyway sorry if that seems a tiny bit direct and rude as i dont want to come arcross that way as i do think it was a very well run course, just probably one i shouldn't have done. Thank you anyway.

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CJS's reflections, 5 years

Surprising consistency and stability.

Each year we have ended up 40 $\pm$ 2 problems from the same place. (35 pages out of 148)

- **Week 1** Anticipation.
- **Week 2** Excitement and enthusiasm.
- **Week 3** Frustration.
- **Week 4-5** Despondency, Doldrums and Despair.
- **Week 6-7** Re-build confidence.
- **Week 8-9** Adjust expectations.
- **Week 10-11** Collegiate conviviality

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Mathematical support

Habits of mind

- playing around;
- always plot the graph;
- a focus on relationships;
- alternative non-algebraic mode.

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But I don’t have time!

Gradually gain confidence with the tool

- Static diagrams for handouts
- Pre-prepared diagrams in lectures with steps
- Starting with a blank sheet...
- Students and problem solving
- Using in the Support Centre.

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Geogebra is is very useful for

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2. use in a lecture,
3. use by students in presentations,
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