Piecewise Defined Functions

Some functions are defined by **different formulas** in different parts of their domains.

Such functions are called **piecewise defined functions**.

Example

The function f is defined by

$$f(x) = \begin{cases} 1 - x & \text{if } x \leqslant -1 \\ x^2 & \text{if } x > -1 \end{cases}$$

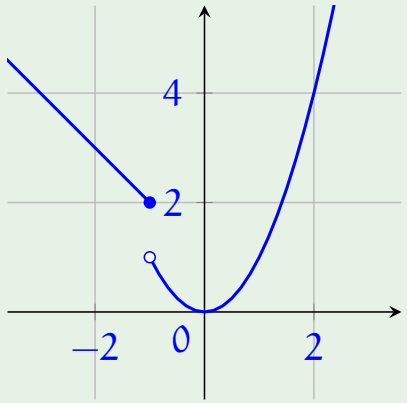
We can read this definition as

- When x is less or equal than -1, then $f(x)$is equal to $1 - x$.
- When x is greater than -1, then $f(x)$is equal to x^2.

Let's calculate some values: in other words, find the y values when given an x value.

- $f(-2) = 1 - (-2) = 3$ since $-2 \leqslant -1$.
- $f(-1) = 1 - (-1) = 2$ since $-1 \leqslant -1$.
- $f(0) = 0^2 = 0$ since $0 > -1$.
- $f(2) = 2^2 = 4$ since $2 > -1$.

You can use these values to sketch the graph of $f(x)$:



Notation: Point $\circ$ **is not** included, whereas point $\bullet$ **is** included.

Exercises

Exercises

Consider these functions:

$$f(x) = \begin{cases} 2x-2 & \text{if } x < 1 \\ -x & \text{if } x \geqslant 1 \end{cases} \qquad g(x) = \begin{cases} 3 & \text{if } x < -2 \\ x+1 & \text{if } -2 \leqslant x < 1 \\ 2x & \text{if } x \geqslant 1 \end{cases}$$

(a) Calculate

(i) $f(0)$, $f(-5)$, $f(1)$, $f(2)$,

(ii) $g(-4)$, $g(-2)$, $g(0)$, $g(1)$, $g(\frac{3}{2})$, $g(3)$.

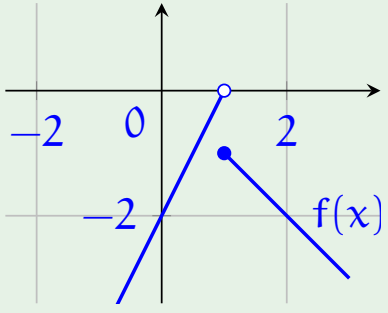
(b) Sketch the graphs of $f(x)$ and $g(x)$.

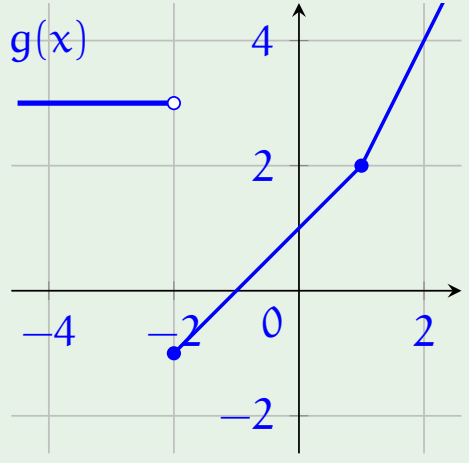
Exercises

Answers

(a) (i) $-2, -12, -1, -2,$
(ii) $3, -1, 1, 2, 3, 6.$

(b)





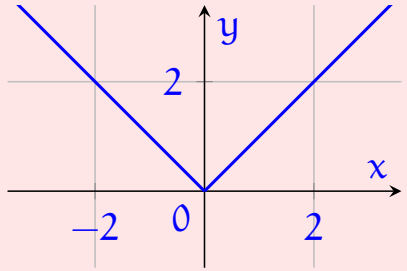
Some Examples: 1. The Absolute Value Function

- The **absolute value** of a number a is denoted by $|a|$.
- Recall that $|a| = a$ if $a \geqslant 0$ and $|a| = -a$ if $a < 0$.

The **absolute value function** $f(x) = |x|$ is defined as

$$|x| = \begin{cases} x, & \text{if } x \geqslant 0, \\ -x, & \text{if } x < 0. \end{cases}$$

Calculate a few values, say $f(1) = |1| = 1$ and $f(-2) = |-2| = 2$, and sketch the graph:



2. The Percentage Grade Function

The **percentage grade** function g is piecewise defined:

$$g(x) = \begin{cases} \mathtt{F} & \text{if } 0 \leqslant x < 30 \\ \mathtt{E-} & \text{if } 30 \leqslant x < 35 \\ \mathtt{E+} & \text{if } 35 \leqslant x < 40 \\ \vdots & \vdots \\ \mathtt{B} & \text{if } 60 \leqslant x < 70 \\ \mathtt{A} & \text{if } 70 \leqslant x \leqslant 100 \end{cases}$$

3. The Heaviside Step Function

The **Heaviside step function**, also called **unit step function**, is being used to represent a signal that switches on at a specified time and stays switched on indefinitely.

The **Heaviside step function** H is defined as

$$H(t) = \begin{cases} 0, & \text{if } t < 0, \\ 1, & \text{if } t \geqslant 0. \end{cases}$$