Natural Numbers

The first type of numbers we come across with are the **natural numbers**. We use them for counting objects, thus they are also called **counting numbers**.

The **natural numbers** denoted $\mathbb{N}$ are the positive whole numbers. We describe them as follows:

$$\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}.$$

Note: You may find different definitions in the literature:

- Some definitions include 0 and use the term *natural numbers* for { 0,1,2,3,4,5,. . . }.
- Natural numbers including 0 are also called **whole numbers**.

Integers

The **integers** denoted $\mathbb{Z}$ are all the natural numbers, zero, and their negatives.

$$\mathbb{Z} = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots, \}$$

Think of

- temperature $-4^o\,C$,
- or money: €300 Overdrawn is really -300!

Rational Numbers

Rational numbers are all the fractions $\frac{a}{b}$, where both a and b are integers, and b can never be equal to 0. More formally:

The **rational numbers** denoted $\mathbb{Q}$ are

$$\mathbb{Q} = \{\frac{a}{b} : a, b \in \mathbb{Z}, \quad b \neq 0\}.$$

For example, $\frac{1}{2} = 0.5$ and $-3 = \frac{-3}{1}$ are rational numbers, and $\frac{1}{3} = 0.\dot{3} = 0.333333333...$ is a rational number.

Terminating decimal numbers such as 0.5 and repeating decimal numbers such as $0.\dot{3} = 0.333333333...$ **are rational numbers**.

Irrational Numbers

Irrational numbers cannot be written in fraction form, i.e. they cannot be written as the ratio of the two integers.

Irrational numbers are numbers on the number line that cannot be written as rational numbers.

Their decimal representation goes on forever without ever settling into a repeating pattern.

Examples of irrational numbers include $\sqrt{2}, \sqrt{3}, \sqrt[3]{4}, \pi$ and the so called *golden ratio*:

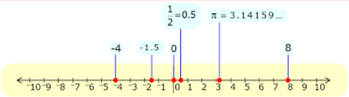
$$\frac{1+\sqrt{5}}{2}.$$

Real Numbers

The numbers we have seen so far are

- The Natural Numbers $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$.
- The Integers $\mathbb{Z} = \{\ldots, -2, -1, -3, 0, 1, 2, 3, \ldots\}$.
- The Rational Numbers $\mathbb{Q} = \{\frac{a}{b} :$ where a, b are integers$\}$.
- The Irrational Numbers.

The **Real Numbers** $\mathbb{R}$ is the set of all of the above numbers. We can think of it as "all numbers" on the number line.



Note: There is another set of numbers called the **Complex Numbers** $\mathbb{C}$ which includes the real numbers.

Real Numbers

Another way to visualise real numbers and to learn how the different types of numbers are related is to represent them in a so called **Venn Diagram**: