Factorising quadratics

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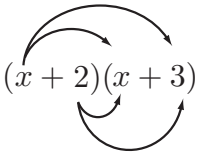
You will have seen before that expressions like $(x+2)(x+3)$ can be expanded to give the quadratic expression x^2+5x+6. Like many processes in mathematics, it is useful to be able to go the other way. That is, starting with the quadratic expression x^2+5x+6, can we carry out a process which will result in the form $(x+2)(x+3)$? This process is called **factorising the quadratic expression**. This leaflet describes this process. Special cases known as **complete squares** and **the difference of two squares** are dealt with on separate leaflets.

Factorising quadratics

To learn how to factorise let us study again the previous example when the brackets were multiplied out from $(x+2)(x+3)$ to give x^2+5x+6.



$$(x+2)(x+3) \quad = x^2+3x \quad + \quad 2x+6 \quad = x^2+5x \; + \; 6$$

Clearly the number 6 in the final answer comes from *multiplying* the numbers 2 and 3 in the brackets. This is an important observation. The term $5x$ comes from *adding* the terms $3x$ and $2x$.

So, if we were to begin with x^2+5x+6 and we were going to reverse the process we need to look for two numbers which add to give 5 and multiply to give 6. What are these numbers ? Well, we know that they are 3 and 2, and you will learn with practice to find these simply by inspection. We can set the calculation out as follows. Start with a pair of empty brackets.

$$\begin{aligned} x^2+5x+6 &= (\quad)(\quad) && \text{insert an } x \text{ in each} \\ &= (x\quad)(x\quad) && \text{these will multiply to give the required } x^2 \\ &= (x+2)(x+3) && \text{these numbers multiply to give 6 and add to give 5} \end{aligned}$$

The answer should always be checked by multiplying-out the brackets again!

Example

Factorise the quadratic expression $x^2-7x+12$.

Starting as before we write

$$x^2-7x+12 = (x\quad)(x\quad)$$

and we look for two numbers which add together to give -7 and which multiply together to give 12. The two numbers we seek are -3 and -4 because

$$-3 \times -4 = 12, \qquad \text{and} \qquad -3 + -4 = -7$$

So

$$x^2-7x+12 = (x-3)(x-4)$$

Once again, note that the answer can be checked by multiplying-out the brackets again. The alternative, equivalent form $(x-4)(x-3)$, is also correct.

Exercises

1. Factorise the following.

a) $x^2 + 8x + 15$ b) $x^2 + 10x + 24$ c) $x^2 + 9x + 8$ d) $x^2 + 9x + 14$
e) $x^2 + 15x + 36$ f) $x^2 + 2x - 3$ g) $x^2 + 2x - 8$ h) $x^2 + x - 20$

Quadratic expressions where the coefficient of x is not 1

Let us try to factorise the expression $3x^2 + 5x - 2$. We write, as before,

$$3x^2 + 5x - 2 = (\quad)(\quad)$$

and try, by inspection, to determine the contents of the brackets. There is no point writing $(x \quad)(x \quad)$ because the two x terms would multiply to give x^2, and in this example we are looking for $3x^2$. So try

$$3x^2 + 5x - 2 = (3x \quad)(x \quad)$$

which will certainly generate the term $3x^2$. The constant term -2 can be generated from the numbers -2 and 1, or alternatively -1 and 2. So, we are led to consider the following combinations

$$(3x - 2)(x + 1), \qquad (3x + 1)(x - 2), \qquad (3x - 1)(x + 2), \qquad (3x + 2)(x - 1)$$

all of which generate the correct term in x^2 and the correct constant term. However, only one of these generates the correct x term, $5x$. By inspection we find

$$3x^2 + 5x - 2 = (3x - 1)(x + 2)$$

Example

Factorise $2x^2 + 5x - 7$.

To generate the term $2x^2$ we can write

$$2x^2 + 5x - 7 = (2x \quad)(x \quad)$$

To generate the constant term -7 we need two numbers which multiply together to give -7. Recognise that to produce a negative result one factor must be positive and one must be negative. We are led to consider -7 and 1, or alternatively -1 and 7. So, we consider the following combinations

$$(2x - 7)(x + 1), \qquad (2x + 1)(x - 7), \qquad (2x - 1)(x + 7), \qquad (2x + 7)(x - 1)$$

By inspection the correct factorisation is $2x^2 + 5x - 7 = (2x + 7)(x - 1)$.

Exercises

2 Factorise the following.

a) $2x^2 + 11x + 5$ b) $3x^2 + 19x + 6$ c) $3x^2 + 17x - 6$ d) $6x^2 + 7x + 2$
e) $7x^2 - 6x - 1$ f) $12x^2 + 7x + 1$ g) $8x^2 + 6x + 1$ h) $8x^2 - 6x + 1$

Answers

1. a) $(x + 3)(x + 5)$ b) $(x + 4)(x + 6)$ c) $(x + 1)(x + 8)$ d) $(x + 2)(x + 7)$
e) $(x + 3)(x + 12)$ f) $(x + 3)(x - 1)$ g) $(x + 4)(x - 2)$ h) $(x + 5)(x - 4)$

2. a) $(2x + 1)(x + 5)$ b) $(3x + 1)(x + 6)$ c) $(3x - 1)(x + 6)$ d) $(2x + 1)(3x + 2)$
e) $(7x + 1)(x - 1)$ f) $(3x + 1)(4x + 1)$ g) $(2x + 1)(4x + 1)$ h) $(2x - 1)(4x - 1)$