Engineering Maths First Aid Kit

2.14

Quadratic equations 1

Introduction

This leaflet will explain how many quadratic equations can be solved by **factorisation**.

1. Quadratic equations

A **quadratic equation** is an equation of the form $ax^2 + bx + c = 0$, where a, b and c are constants. For example $3x^2 + 2x - 9 = 0$ is a quadratic equation with $a = 3$, $b = 2$ and $c = -9$.

The constants b and c can have any value including 0. The constant a can have any value except 0. This is to ensure that the equation has an x^2 term. We often refer to a as the coefficient of x^2, to b as the coefficient of x and to c as the constant term. Usually, a, b and c are known numbers, whilst x represents an unknown quantity which we will be trying to find.

2. The solutions of a quadratic equation

To **solve** a quadratic equation we must find values for x which when substituted into the equation make the left-hand and right-hand sides equal. These values are also called **roots**. For example the value $x = 4$ is a solution of the equation $x^2 - 3x - 4 = 0$ because substituting 4 for x we find

$$4^2 - 3(4) - 4 = 16 - 12 - 4$$

which simplifies to zero, the same as the right-hand side of the equation. There are several techniques which can be used to solve quadratic equations. One of these, *factorisation*, is discussed in this leaflet. You should be aware that not all quadratic equations can be solved by this method. An alternative method which uses a formula is described on leaflet 2.15.

3. Solving a quadratic equation by factorisation.

Sometimes, but not always, it is possible to solve a quadratic equation using factorisation. If you need to revise factorisation you should see leaflet *2.6 Factorising quadratics*.

Example
Solve the equation $x^2 + 7x + 12 = 0$ by factorisation.

Solution
We first factorise $x^2 + 7x + 12$ as $(x + 3)(x + 4)$. Then the equation becomes $(x + 3)(x + 4) = 0$.

It is important that you realise that if the product of two quantities is zero, then one or both of the quantities must be zero. It follows that either

$$x + 3 = 0, \text{ that is } x = -3 \qquad \text{or} \qquad x + 4 = 0, \text{ that is } x = -4$$

The roots of $x^2 + 7x + 12 = 0$ are $x = -3$ and $x = -4$.

Example
Solve the quadratic equation $x^2 + 4x - 21 = 0$.

Solution
$x^2 + 4x - 21$ can be factorised as $(x + 7)(x - 3)$. Then

$$\begin{aligned} x^2 + 4x - 21 &= 0 \\ (x + 7)(x - 3) &= 0 \end{aligned}$$

Then either

$$x + 7 = 0, \text{ that is } x = -7 \qquad \text{or} \qquad x - 3 = 0, \text{ that is } x = 3$$

The root of $x^2 + 4x - 21 = 0$ are $x = -7$ and $x = 3$.

Example
Find the roots of the quadratic equation $x^2 - 10x + 25 = 0$.

Solution

$$x^2 - 10 + 25 = (x - 5)(x - 5) = (x - 5)^2$$

Then

$$\begin{aligned} x^2 - 10x + 25 &= 0 \\ (x - 5)^2 &= 0 \\ x &= 5 \end{aligned}$$

There is one root, $x = 5$. Such a root is called a **repeated root**.

Example
Solve the quadratic equation $2x^2 + 3x - 2 = 0$.

Solution
The equation is factorised to give

$$(2x - 1)(x + 2) = 0$$

so, from $2x - 1 = 0$ we find $2x = 1$, that is $x = \frac{1}{2}$. From $x + 2 = 0$ we find $x = -2$. The two solutions are therefore $x = \frac{1}{2}$ and $x = -2$.

Exercises
1. Solve the following quadratic equations by factorization.

a) $x^2 + 7x + 6 = 0$, b) $x^2 - 8x + 15 = 0$, c) $x^2 - 9x + 14 = 0$,

d) $2x^2 - 5x - 3 = 0$, e) $6x^2 - 11x - 10 = 0$, f) $6x^2 + 13x + 6 = 0$.

Answers
a) $-1, -6$, b) $3, 5$, c) $2, 7$, d) $3, -\frac{1}{2}$, e) $\frac{5}{2}, -\frac{2}{3}$, f) $x = -\frac{3}{2}, x = -\frac{2}{3}$.