

# What is a function ?

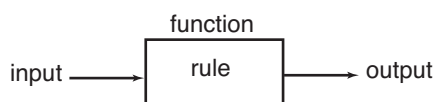
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## Introduction

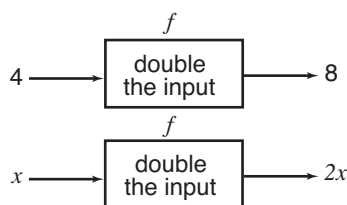
A quantity whose value can change is known as a **variable**. **Functions** are used to describe the rules which define the ways in which such a change can occur. The purpose of this leaflet is to explain functions and their notation.

## The function rule

A function is a rule which operates on an **input** and produces an **output**. This can be illustrated using a **block diagram** such as that shown below. We can think of the function as a mathematical machine which processes the input, using a given rule, in order to produce an output. We often write the rule inside the box.



In order for a rule to be a function it must produce only a single output for any given input. The function with the rule 'double the input' is shown below.



Note that with an input of 4 the function would produce an output of 8. With a more general input,  $x$  say, the output will be  $2x$ . It is usual to assign a letter or other symbol to a function in order to label it. The doubling function pictured in the example above has been given the symbol  $f$ .

A function is a rule which operates on an input and produces a single output from that input.

For the doubling function it is common to use the notation

$$f(x) = 2x$$

This indicates that with an input  $x$ , the function,  $f$ , produces an output of  $2x$ . The input to the function is placed in the brackets after the function label ' $f$ '.  $f(x)$  is read as ' $f$  is a function of  $x$ ', or simply ' $f$  of  $x$ ', meaning that the output from the function depends upon the value of the input  $x$ .

## Example

State the rule of each of the following functions:

- a)  $f(x) = 7x + 9$ ,   b)  $h(t) = t^3 + 2$ ,   c)  $p(x) = x^3 + 2$ .

### Solution

- a) The rule for  $f$  is 'multiply the input by 7 and then add 9'.
- b) The rule for  $h$  is 'cube the input and add 2'.
- c) The rule for  $p$  is 'cube the input and add 2'.

Note from parts b) and c) that it is the rule that is important when describing a function and not the letters being used. Both  $h(t)$  and  $p(x)$  instruct us to 'cube the input and add 2'.

The input to a function is called its **argument**. We can obtain the output from a function if we are given its argument. For example, given the function  $f(x) = 3x + 2$  we may require the value of the output when the argument is 5. We write this as  $f(5)$ . Here,  $f(5) = 3 \times 5 + 2 = 17$ .

### Example

Given the function  $f(x) = 4x + 3$  find a)  $f(-1)$ ,      b)  $f(6)$

### Solution

- a) Here the argument is  $-1$ . We find  $f(-1) = 4 \times (-1) + 3 = -1$ .
- b)  $f(6) = 4(6) + 3 = 27$ .

Sometimes the argument will be an algebraic expression, as in the following example.

### Example

Given the function  $y(x) = 5x - 3$  find

- a)  $y(t)$ ,      b)  $y(7t)$ ,      c)  $y(z + 2)$ .

### Solution

The function rule is multiply the input by 5, and subtract 3. We can apply this rule whatever the argument.

- a) To find  $y(t)$  multiply the argument,  $t$ , by 5 and subtract 3 to give  $y(t) = 5t - 3$ .
- b) Now the argument is  $7t$ . So  $y(7t) = 5(7t) - 3 = 35t - 3$ .
- c) In this case the argument is  $z + 2$ . We find  $y(z + 2) = 5(z + 2) - 3 = 5z + 10 - 3 = 5z + 7$ .

### Exercises

1. Write down a function which can be used to describe the following rules:
  - a) 'cube the input and divide the result by 2',      b) 'divide the input by 5 and then add 7'
2. Given the function  $f(x) = 7x - 3$  find a)  $f(3)$ ,      b)  $f(6)$ ,      c)  $f(-2)$ .
3. If  $g(t) = t^2$  write down expressions for a)  $g(x)$ ,      b)  $g(3t)$ ,      c)  $g(x + 4)$ .

### Answers

1. a)  $f(x) = \frac{x^3}{2}$ ,      b)  $f(x) = \frac{x}{5} + 7$ .      2. a) 18,      b) 39,      c)  $-17$
3. a)  $g(x) = x^2$ ,      b)  $g(3t) = (3t)^2 = 9t^2$ ,      c)  $g(x + 4) = (x + 4)^2 = x^2 + 8x + 16$ .