Logarithms - changing the base

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Sometimes it is necessary to find logarithms to bases other than 10 and e. For example, logarithms to the base 2 are used in communications engineering. Your calculator can still be used but you need to apply a formula for changing the base. This leaflet gives this formula and shows how to use it.

A formula for change of base

Suppose we want to calculate a logarithm to base 2. The formula states

$$\log_2 x = \frac{\log_{10} x}{\log_{10} 2}$$

So we can calculate base 2 logarithms using base 10 logarithms obtained using a calculator.

Examples

$$\log_2 36 = \frac{\log_{10} 36}{\log_{10} 2} = \frac{1.556303}{0.301030} = 5.1699 \text{ (correct to 4 d.p.)}$$

$$\log_2 64 = \frac{\log_{10} 64}{\log_{10} 2} = \frac{1.806180}{0.301030} = 6$$

Check these for yourself. More generally, for bases a and b,

$$\log_a x = \frac{\log_b x}{\log_b a}$$

In particular, by choosing $b = 10$ we find

$$\log_a x = \frac{\log_{10} x}{\log_{10} a}$$

Use this formula to check that $\log_{20} 100 = 1.5372$ (correct to 4 d.p.).

Exercises

1. Find, correct to 3 decimal places, (a) $\log_2 15$, (b) $\log_2 56.25$, (c) $\log_3 16$.

2. By writing the expression in logarithmic form, find the number x such that $2^x = 3.6$.

Answers

1. (a) 3.907 (3 d.p.), (b) 5.814 (3 d.p.), (c) 2.524 (3 d.p).
2. $\log_2 3.6 = x$, and so $x = 1.848$ (3 d.p.).