

# Engineering Maths

## First Aid Kit

4.6

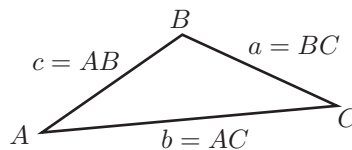
## The sine rule and cosine rule

### Introduction

To **solve** a triangle is to find the lengths of each of its sides and all its angles. The **sine rule** is used when we are given either a) two angles and one side, or b) two sides and a non-included angle. The **cosine rule** is used when we are given either a) three sides or b) two sides and the included angle.

### 1. The sine rule

Study the triangle  $ABC$  shown below. Let  $B$  stand for the angle at  $B$ . Let  $C$  stand for the angle at  $C$  and so on. Also, let  $b = AC$ ,  $a = BC$  and  $c = AB$ .



The sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

### Example

In triangle  $ABC$ ,  $B = 21^\circ$ ,  $C = 46^\circ$  and  $AB = 9\text{cm}$ . Solve this triangle.

### Solution

We are given two angles and one side and so the sine rule can be used. Furthermore, since the angles in any triangle must add up to  $180^\circ$  then angle  $A$  must be  $113^\circ$ . We know that  $c = AB = 9$ . Using the sine rule

$$\frac{a}{\sin 113^\circ} = \frac{b}{\sin 21^\circ} = \frac{9}{\sin 46^\circ}$$

So,

$$\frac{b}{\sin 21^\circ} = \frac{9}{\sin 46^\circ}$$

from which

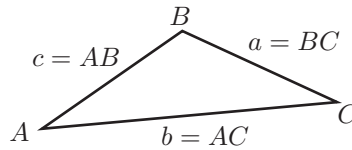
$$b = \sin 21^\circ \times \frac{9}{\sin 46^\circ} = 4.484\text{cm.} \quad (3\text{dp})$$

Similarly

$$a = \sin 113^\circ \times \frac{9}{\sin 46^\circ} = 11.517\text{cm.} \quad (3\text{dp})$$

## 2. The cosine rule

Refer to the triangle shown below.



The cosine rule:

$$a^2 = b^2 + c^2 - 2bc \cos A, \quad b^2 = a^2 + c^2 - 2ac \cos B, \quad c^2 = a^2 + b^2 - 2ab \cos C$$

### Example

In triangle  $ABC$ ,  $AB = 42\text{cm}$ ,  $BC = 37\text{cm}$  and  $AC = 26\text{cm}$ . Solve this triangle.

### Solution

We are given three sides of the triangle and so the cosine rule can be used. Writing  $a = 37$ ,  $b = 26$  and  $c = 42$  we have

$$a^2 = b^2 + c^2 - 2bc \cos A$$

from which

$$37^2 = 26^2 + 42^2 - 2(26)(42) \cos A$$
$$\cos A = \frac{26^2 + 42^2 - 37^2}{(2)(26)(42)} = \frac{1071}{2184} = 0.4904$$

and so

$$A = \cos^{-1} 0.4904 = 60.63^\circ$$

You should apply the same technique to verify that  $B = 37.76^\circ$  and  $C = 81.61^\circ$ . You should also check that the angles you obtain add up to  $180^\circ$ .

### Exercises

1. Solve the triangle  $ABC$  in which  $AC = 105\text{cm}$ ,  $AB = 76\text{cm}$  and  $A = 29^\circ$ .
2. Solve the triangle  $ABC$  given  $C = 40^\circ$ ,  $b = 23\text{cm}$  and  $c = 19\text{cm}$ .

### Answers

1.  $a = 53.31\text{cm}$ ,  $B = 107.28^\circ$ ,  $C = 43.72^\circ$ .
2.  $A = 11.09^\circ$ ,  $B = 128.91^\circ$ ,  $a = 5.69\text{cm}$ .